Örnek: Aşağıdaki matrislerin ($\mathbb{R}$ üzerinde) değişmez çarpanlarını ve temel bölenlerini bulunuz.

a) $A=\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} \longrightarrow (xI-A)$ Smith Normal F.

$$xI-A=\begin{bmatrix} x & +2 \\ -1 & x \end{bmatrix} \overset{-S_2 \Leftrightarrow S_1}{\sim} \begin{bmatrix} 1 & -x \\ x & 2 \end{bmatrix}$$

$$\begin{bmatrix} d_1 & & \\ & d_2 & \\ & & \ddots \\ & & & d_s \end{bmatrix}$$

$$\overset{-x.S_1+S_2}{\sim} \begin{bmatrix} 1 & -x \\ 0 & x^2+2 \end{bmatrix} \sim \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & x^2+2 \end{bmatrix}}_{S.N.F}$$

$d_i \mid d_{i+1}$

$d_1=1$, $d_2=x^2+2$ (Değşm. Çarp.)

Temel Bölenler: x^2+2

$\boxed{\text{Temel Bölenler ($\mathbb{C}$'de): } x-\sqrt{2}\,i,\ x+\sqrt{2}\,i}$

$$R(A)=\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$$

$$R_A(A)=\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$$

$$J(A)=\begin{bmatrix} -\sqrt{2}\,i & 0 \\ 0 & \sqrt{2}\,i \end{bmatrix}$$

b) $A=\begin{bmatrix}0 & -2\\ -1 & 0\end{bmatrix}$; $xI-A=\begin{bmatrix}x & 2\\ 1 & x\end{bmatrix}$

$$(xI-A)\overset{S_1\leftrightarrow S_2}{\sim}\begin{bmatrix}1 & x\\ x & 2\end{bmatrix}\overset{-x.S_1+S_2}{\sim}\begin{bmatrix}1 & x\\ 0 & 2-x^2\end{bmatrix}\sim\cdots\underbrace{\begin{bmatrix}1 & 0\\ 0 & x^2-2\end{bmatrix}}$$

Değişmez Çarpanlar: $d_1=1$, $d_2=x^2-2$

Temel Bölenler: $x-\sqrt{2}$, $x+\sqrt{2}$

$$R(A)=\begin{bmatrix}0 & 2\\ 1 & 0\end{bmatrix}\leftarrow$$

$$R_A(A)=\left[\begin{array}{c|c}\sqrt{2} & 0\\ \hline 0 & -\sqrt{2}\end{array}\right]$$

$$J(A)=\begin{bmatrix}\sqrt{2} & 0\\ 0 & -\sqrt{2}\end{bmatrix}$$

c) $A = \text{köşeg}[-1, -1, 0, -1, 5, -3]$

$$xI - A = \begin{bmatrix} x+1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$C_3 + C_1 \to C_1$

$\sim$

$a(x).p(x) + b(x).q(x) = d(x)$

$1.(1+x) + (-1).x = 1$

ebob

$$\sim \begin{bmatrix} x+1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$-S_3 + S_1$

$$\sim \begin{bmatrix} 1 & 0 & -x & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$$-xS_1+S_3 \sim \begin{bmatrix} 1 & 0 & -x & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1 & 0 & -x & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}}$$

$$\begin{bmatrix} 1&0&0&0&0&0\\ 0&x+1&0&0&0&0\\ 0&0&x^2+x&0&0&0\\ 0&0&0&x+1&0&0\\ 0&0&0&0&x-5&0\\ 0&0&0&0&0&x+3 \end{bmatrix} \underset{-C_5+C_2}{\sim} \begin{bmatrix} 1&0&0&0&0&0\\ 0&x+1&0&0&0&0\\ 0&0&x^2+x&0&0&0\\ 0&0&0&x+1&0&0\\ 0&5-x&0&0&x-5&0\\ 0&0&0&0&0&x+3 \end{bmatrix}$$

$$\underset{S_5+S_2}{\sim} \begin{bmatrix} 1&0&0&0&0&0\\ 0&6&0&0&x-5&0\\ 0&0&x^2+x&0&0&0\\ 0&0&0&x+1&0&0\\ 0&5-x&0&0&x-5&0\\ 0&0&0&0&0&x+3 \end{bmatrix} \frac{1}{6}\cdot C_2 \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&x-5&0\\ 0&0&x^2+x&0&0&0\\ 0&0&0&x+1&0&0\\ 0&\frac{5-x}{6}&0&0&x-5&0\\ 0&0&0&0&0&x+3 \end{bmatrix}$$

$$\underset{\frac{x-5}{6}\cdot S_2+S_5}{\sim} \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&x-5&0\\ 0&0&x^2+x&0&0&0\\ 0&0&0&x+1&0&0\\ 0&0&0&0&z_1&0\\ 0&0&0&0&0&x+3 \end{bmatrix}$$

$$z_1 = \frac{(x-5)^2+6(x-5)}{6}$$

$$z_1 = \frac{x^2-4x-5}{6} = \frac{(x-5)(x+1)}{6}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & x-5 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$$z_1' = (x-5)(x+1)$$

$$\underset{\sim}{-C_6+C_4} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & -x-3 & 0 & x+3 \end{bmatrix} \overset{S_6+S_4}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & -x-3 & 0 & x+3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & -x-3 & 0 & x+3 \end{bmatrix} \overset{-\frac{1}{2}C_4}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & \frac{x+3}{2} & 0 & x+3 \end{bmatrix}$$

$$\overset{-\frac{(x+3)}{2}S_4+S_6}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_2 \end{bmatrix}$$

$$z_2 = \frac{-(x+3)^2 + 2(x+3)}{2}$$

$$z_2 = -\frac{(x^2+4x+3)}{2}$$

$$z_2' = (x+3)(x+1)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_2' \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_2' \end{bmatrix}$$

$$z_2' = (x+3)(x+1)$$

$$z_1' = (x-3)(x+1)$$

$$a.(x+3)(x+1) + b.(x+1)x = (x+1)$$

$$x^2(a+b) + (4a+b).x + 3a = x+1$$

$$a = \frac{1}{3}, \quad b = -\frac{1}{3}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x.(x+1) & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_2' \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x\cdot(x+1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2i & 0 \\ 0 & 0 & 0 & 0 & 0 & 22 \end{bmatrix}$$

$\frac{1}{3}C_6 + C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-x(x+1)}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2i & 0 \\ 0 & 0 & 0 & \frac{1}{3}(x+3)(x+1) & 0 & (x+3)(x+1) \end{bmatrix}$$

$-\frac{1}{3}S_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-x(x+1)}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & \frac{1}{3}(x+3)(x+1) & 0 & (x+3)(x+1) \end{bmatrix}$$

$$S_6 + S_4 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (x+1) & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & \frac{(x+3)(x+1)}{3} & 0 & (x+3)(x+1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (x+1) & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & \frac{(x+3)(x+1)}{3} & 0 & (x+3)(x+1) \end{bmatrix} \quad 3.S_6 \quad \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (x+1) & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & (x+1)(x+3) & 0 & 3(x+3)(x+1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (x+1) & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & (x+1)(x+3) & 0 & 3(x+3)(x+1) \end{bmatrix} \quad -(x+3).S_4+S_6$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_3 \end{bmatrix}$$

$$z_3 = (x+3)(x+1)\left[3-x-3\right]$$

$$z_3 = -x.(x+3)(x+1)$$

$$z_3' = x.(x+3)(x+1)$$

$$z_1' = (x-5)(x+1)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_3 \end{bmatrix}$$

$$z_3 = (x+3)(x+1)\left[3-x-3\right]$$

$$z_3 = -x.(x+3)(x+1)$$

$$z_3' = x.(x+3)(x+1)$$

$$z_1' = (x-5)(x+1)$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_3' \end{bmatrix}$$

$$\underbrace{a(x^3+4x^2+3x)}_{z_3'} + \underbrace{b(x^2-4x-5)}_{z_1'} = (x+1)$$

$$a.(x^3+4x^2+3x) + (b_1x+b_2).(x^2-4x-5) = (x+1)$$

$$(a+b_1).x^3 + x^2.(4a-4b_1+b_2) + x.(3a-5b_1-4b_2) - 5b_2 = (x+1)$$

$a=-b_1$, $b_2 = -\frac{1}{5}$, $-8b_1 - \frac{1}{5} = 0$, $b_1 = \frac{1}{40}$

$\frac{3}{40} + \frac{5}{40} = \frac{1}{5}$ $+\frac{4}{5} = 1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_3' \end{bmatrix}$$

$a(x^3+4x^2+3x) + b(x^2-4x-5) = (x+1)$

$a.(x^3+4x^2+3x) + (b_1 x+b_2).(x^2-4x-5) = (x$

$(a+b_1).x^3 + x^2.(4a-4b_1+b_2) + x.(3a-5.b_1-4b_2) - 5b_2 = (x+1)$

$a = -b_1, \quad b_2 = -\frac{1}{5}, \quad -8b_1 - \frac{1}{5} = 0, \quad b_1 = -\frac{1}{40}$

$\frac{1}{40}.C_6 + C_5$

$\sim$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{40} z_3' & z_3' \end{bmatrix}$$

$a=-b_1$, $b_2=-\frac{1}{5}$, $-8b_1-\frac{1}{5}=0$, $b_1=\frac{1}{40}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{40}z_3' & z_3' \end{bmatrix}$$

b. S_5+S_6

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & x+1 & z_3' \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_i' & 0 \\ 0 & 0 & 0 & 0 & x+1 & z_3' \end{bmatrix}$$

$(5-x) \cdot S_6 + S_5$

$\sim$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (5-x) \cdot z_3' \\ 0 & 0 & 0 & 0 & x+1 & z_3' \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (5-x).z_3' \\ 0 & 0 & 0 & 0 & x+1 & z_3' \end{bmatrix}$$

$S_5 \leftrightarrow S_6$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & (x+1) & z_3' \to 0 \\ 0 & 0 & 0 & 0 & 0 & (x-5)(x+1).(x+3).x \end{bmatrix}$$

$|xI-A|_{6\times 6}$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & x+1 & 0 & 0 \\
0 & 0 & 0 & 0 & (x+1) & z_3 \rightarrow 0 \\
0 & 0 & 0 & 0 & 0 & (x-5)(x+1).(x+3).x
\end{bmatrix}
$$

Değişmez Çarpanlar: $d_1 = 1$, $d_2 = 1$, $d_3 = 1$

$\underline{d_4 = x+1}$, $\underline{d_5 = x+1}$,

$d_6 = x.(x+1).(x-5)(x+3)$

$= x^4 + a_3 x^3 + a_2 . x^2 + a_1 x + a_0$

Temel Bölenler: $x+1$, $x+1$, x, $x+1$, $x-5$, $x+3$

$$
R(A) = \left[\begin{array}{c|c|cccc}
-1 & & & & & \\ \hline
 & -1 & & & & \\ \hline
 & & 0 & 0 & 0 & -a_0 \\
 & & 1 & 0 & 0 & -a_1 \\
 & & 0 & 1 & 0 & -a_2 \\
 & & 0 & 0 & 1 & -a_3
\end{array}\right]
$$

$$
R_A(A) = \left[\begin{array}{c|c|c|c|c|c}
-1 & & & & & \\ \hline
 & -1 & & & & \\ \hline
 & & -1 & & & \\ \hline
 & & & 0 & & \\ \hline
 & & & & -3 & \\ \hline
 & & & & & 5
\end{array}\right] = J(A)
$$

Örnek: $\Delta_A = |xI - A| = (x+2)^3 \cdot (x-2)^4$

ve $\min_A(x) = (x+2)^2 (x-2)^2$ olan A

matrisinin tüm kanonik formlarını bulunuz.

$A_{7\times 7}$

$$(xI - A) \sim \begin{bmatrix} d_1 & & 0 \\ & \ddots & \\ 0 & & d_7 \end{bmatrix}$$

$|xI - A| = d_1 \cdots d_7$

$=$

$\min_A(x) = d_7 = (x+2)^2 \cdot (x-2)^2$

$d_6 = (x-2)^2 \cdot (x+2)$

$d_1 = \cdots = d_5 = 1$

(1). durum

$d_6 = (x+2) \cdot (x-2)$

$d_5 = (x-2)$

$d_1 = \cdots = d_4 = 1$

(2). durum

$d_6 = (x+2).(x-2)$

$d_5 = (x-2)$ $\qquad (2)$

$d_1 = \dots = d_4 = 1$

$d_7 = (x-2)^2 (x+2)^2$

$x^4 - 8x^2 + 16$

$$R(A) = \left[\begin{array}{c|cc|cccc} 2 & & & & & & \\ \hline & 0 & 4 & & & & \\ & 1 & 0 & & & & \\ \hline & & & 0 & 0 & 0 & -16 \\ & & & 1 & 0 & 0 & 0 \\ & & & 0 & 1 & 0 & 8 \\ & & & 0 & 0 & 1 & 0 \end{array}\right]$$

(2. durum)

Temel Bölenler: $(x-2), (x+2), (x-2), (x-2)^2, (x+2)^2$

Temel Bölenler: $(x-2), (x+2), (x-2), (x-2)^2, (x+2)^2$

$$R_A(A) = \left[\begin{array}{c|c|c|cc|cc}
2 & & & & & & \\
\hline
 & -2 & & & & & \\
\hline
 & & 2 & & & & \\
\hline
 & & & 0 & -4 & & \\
 & & & 1 & 4 & & \\
\hline
 & & & & & 0 & -4 \\
 & & & & & 1 & -4
\end{array}\right]$$

(2. durum)

Temel Bölenler: $(x-2), (x+2), (x-2), (x-2)^2, (x+2)^2$

$$J(A) = \begin{bmatrix} 2 & & & & & & \\ & -2 & & & & & \\ & & 2 & & & & \\ & & & 2 & 0 & & \\ & & & 1 & 2 & & \\ & & & & & -2 & 0 \\ & & & & & 1 & -2 \end{bmatrix}$$

(2. durum)

$d_6 = (x-2)^2 \cdot (x+2)$, $\underline{d_7 = (x-2)^2 (x+2)^2}$

(1)

$d_1 = \cdots = d_5 = 1$

$d_7 = (x^2 - 4)^2$

$= x^4 - 8x^2 + 16$

$$R(A) = \left[\begin{array}{ccc|cccc} 0 & 0 & -8 & & & & \\ 1 & 0 & 4 & & 0 & & \\ 0 & 1 & 2 & & & & \\ \hline & & & 0 & 0 & 0 & -16 \\ & & & 1 & 0 & 0 & 0 \\ & 0 & & 0 & 1 & 0 & 8 \\ & & & 0 & 0 & 1 & 0 \end{array}\right]$$

$$= \begin{bmatrix} C_{d_6} & \\ & C_{d_7} \end{bmatrix}$$

$d_6 = (x-2)(x^2-4)$

$= x^3 - 2x^2 - 4x + 8$

Temel Bölenler: $(x-2)^2, (x+2), (x-2)^2, (x+2)^2$

1. durum

x^2-2x+4

Temel Bölenler: $(x-2)^2,\ (x+2),\ (x-2)^2,\ (x+2)^2$

$$R_A(A) = \left[\begin{array}{cc|c|cc|cc} 0 & -4 & & & & & \\ 1 & 4 & & & & & \\ \hline & & -2 & & & & \\ \hline & & & 0 & -4 & & \\ & & & 1 & 4 & & \\ \hline & & & & & 0 & -4 \\ & & & & & 1 & -4 \end{array}\right]$$

$J_c^{(m)}$

$$J(A) = \left[\begin{array}{cc|c|cc|cc} 2 & 0 & & & & & \\ 1 & 2 & & & & & \\ \hline & & -2 & & & & \\ \hline & & & 2 & 0 & & \\ & & & 1 & 2 & & \\ \hline & & & & & -2 & 0 \\ & & & & & 1 & -2 \end{array}\right]$$

(1. durum)

$$J_c^{(m)} = \begin{bmatrix} c & & & & 0 \\ 1 & c & & & \\ 0 & 1 & c & \ddots & \\ 0 & 0 & & \ddots & \\ 0 & 0 & \cdots & 1 & c \end{bmatrix}_{m \times m}$$

Örnek: $\underbrace{(x^2-x+1)}_{d_5}$ $\underbrace{(x-3)^2\cdot(x^2-x+1)}_{d_6}$

$$d_6 = (x^2-6x+9)(x^2-x+1) = x^4-7x^3+16x^2-15x+9$$

$d_1 = \cdots = d_4 = 1$

Rasyonel Form:

$$\left[\begin{array}{cccc|cc} 0 & 0 & 0 & -9 & & \\ 1 & 0 & 0 & 15 & & \\ 0 & 1 & 0 & -16 & & 0 \\ 0 & 0 & 1 & 7 & & \\ \hline & & & & 0 & -1 \\ & 0 & & & 1 & 1 \end{array}\right]$$

Temel Bölenler:

$(x^2-x+1), (x^2-x+1), (x-3)^2$

$$(x^2-x+1),\ (x^2-x+1),\ (x-3)^2$$

$$R_A(A) = \begin{bmatrix} 0 & -1 & & & & \\ 1 & 1 & & & & \\ & & 0 & -1 & & \\ & & 1 & 1 & & \\ & & & & 0 & -9 \\ & & & & 1 & 6 \end{bmatrix}$$

$\mathbb{R}$ üzerinde

Jordan Form Yoktur: x^2-x+1 lineer polinomların çarpımı haline getirilemiyor.