Örnek: Aşağıdaki matrislerin ($\mathbb{R}$ üzerinde) değişmez çarpanlarını ve temel bölenlerini bulunuz.

a) $A = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$

$$xI - A = \begin{bmatrix} x & +2 \\ -1 & x \end{bmatrix} \overset{-S_2 \leftrightarrow S_1}{\sim} \begin{bmatrix} 1 & -x \\ x & 2 \end{bmatrix}$$

$$\overset{-x.S_1 + S_2}{\sim} \begin{bmatrix} 1 & -x \\ 0 & x^2+2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & x^2+2 \end{bmatrix}$$

$d_1 = 1$, $d_2 = x^2+2$

Temel Bölenler: x^2+2

Temel Bölenler ($\mathbb{C}$'de): $x-\sqrt{2}\,i$, $x+\sqrt{2}\,i$

$$R(A) = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$$

$$R_A(A) = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$$

$$J(A) = \begin{bmatrix} -\sqrt{2}\,i & 0 \\ 0 & \sqrt{2}\,i \end{bmatrix}$$

b) $A=\begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}$; $xI-A=\begin{bmatrix} x & 2 \\ 1 & x \end{bmatrix}$

$(xI-A) \overset{S_1 \leftrightarrow S_2}{\sim} \begin{bmatrix} 1 & x \\ x & 2 \end{bmatrix} \overset{-x \cdot S_1 + S_2}{\sim} \begin{bmatrix} 1 & x \\ 0 & 2-x^2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & x^2-2 \end{bmatrix}$

Değişmez Çarpanlar: $d_1=1$, $d_2=x^2-2$

Temel Bölenler: $x-\sqrt{2}$, $x+\sqrt{2}$

$R(A)=\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

$R_A(A)=\begin{bmatrix} \sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{bmatrix}$

$J(A)=\begin{bmatrix} \sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{bmatrix}$

c) $A = \text{kőseg}[-1,-1,0,-1,5,-3]$

$$xI-A=\begin{bmatrix} x+1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix} \begin{matrix} C_3+C_1 \to C_1 \\ \sim \end{matrix}$$

$$\sim \begin{bmatrix} x+1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix} \begin{matrix} -S_3+S_1 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & -x & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$-xS_1+S_2$

$\sim$

$$\begin{bmatrix} 1 & 0 & -x & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -x & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ x & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix} \xrightarrow{-C_1+C_2} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 5-x & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$$\xrightarrow{S_5+S_2} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & x-5 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 5-x & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix} \xrightarrow{\frac{1}{6}\cdot C_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & x-5 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & \frac{5-x}{6} & 0 & 0 & x-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$$\xrightarrow{\frac{x-5}{6}\cdot S_2+S_5} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & x-5 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$$z_1 = \frac{(x-5)^2+6(x-5)}{6}$$

$$z_1 = \frac{x^2-4x-5}{6} - \frac{(x-5)(x+1)}{6}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & x-5 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & x+3 \end{bmatrix}$$

$z_1' = (x-5)(x+1)$

$$-C_6+C_4 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & -x-3 & 0 & x+3 \end{bmatrix}$$

S_6+S_4

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & -x-3 & 0 & x+3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & -x-3 & 0 & x+3 \end{bmatrix} \xrightarrow{-\frac{1}{2}C_4} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & \frac{x+3}{2} & 0 & x+3 \end{bmatrix}$$

$$\xrightarrow{-\frac{(x+3)}{2}S_4+S_6} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & z_2 \end{bmatrix}$$

$$z_2 = -\frac{(x+3)^2 + 2(x+3)}{2}$$

$$z_2 = -\frac{(x^2+4x+3)}{2}$$

$$z_2 = (x+3)(x+1)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & x+3 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & x^2+x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_2' \end{bmatrix}$$

$z_2' = (x+3)(x+1)$

$z_1' = (x-3)(x+1)$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x.(x+1) & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_2' \end{bmatrix}$$

$a.(x+3)(x+1) + b.(x+1)x$

$= (x+1)$

$x^2(a+b) + (4a+b)x + 3a$

$= x+1$

$a = \frac{1}{3}$, $b = \frac{-1}{3}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x\cdot(x+1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2i & 0 \\ 0 & 0 & 0 & 0 & 0 & 2i \end{bmatrix} -$$

$$\frac{1}{3}C_6+C_4 \quad \sim \quad -\frac{1}{3}S_4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-x(x+1)}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2i & 0 \\ 0 & 0 & 0 & \frac{1}{3}(x+3)(x+1) & 0 & (x+3)(x+1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-x(x+1)}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & \frac{1}{3}(x+3)(x+1) & 0 & (x+3)(x+1) \end{bmatrix}$$

$$S_6 + S_4 \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (x+1) & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & \frac{(x+3)(x+1)}{3} & 0 & (x+3)(x+1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (x+1) & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & 2_1' & 0 \\ 0 & 0 & 0 & \frac{(x+3)(x+1)}{3} & 0 & (x+3)(x+1) \end{bmatrix} \quad 3 \cdot S_6 \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (x+1) & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & 2_1' & 0 \\ 0 & 0 & 0 & (x+1)(x+3) & 0 & 3(x+3)(x+1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (x+1) & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & (x+1)(x+3) & 0 & 3(x+3)(x+1) \end{bmatrix} \quad -(x+3)\cdot S_4 + S_6$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & 0 & z_3 \end{bmatrix}$$

$$z_3 = (x+3)(x+1)\left[3-x-3\right]$$

$$z_3 = -x\cdot(x+3)(x+1)$$

$$z_3' = x\cdot(x+3)(x+1)$$

$$z_1' = (x-5)(x+1)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & (x+3)(x+1) \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & z_3 \end{bmatrix}$$

$z_3 = (x+3)(x+1) \mid 3-x-3$

$z_3 = -x.(x+3)(x+1)$

$z_3' = x.(x+3)(x+1)$

$z_1' = (x-5)(x+1)$

$a(x^3+4x^2+3x)$

$+b(x^2-4x-5) = (x+1)$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & z_3 \end{bmatrix}$$

$a.(x^3+4x^2+3x) +$

$(b_1x+b_2).(x^2-4x-5) = (x+1)$

$(a+b_1).x^3 + x^2.(4a-4b_1+b_2)$

$+x.(3a-5.b_1-4b_2)-5b_2$

$= (x+1)$

$\frac{3}{40} \cdot \frac{5}{8} = \frac{1}{5} \cdot \frac{5}{5} = 1$

$a = -b_1$, $b_2 = -\frac{1}{5}$, $-8b_1 - \frac{1}{5} = 0$, $b_1 = \frac{1}{40}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & z_3 \end{bmatrix}$$

$a(x^3+4x^2+3x)$
$+b(x^2-4x-5) = (x+1)$

$a.(x^3+4x^2+3x) +$
$(b_1x+b_2).(x^2-4x-5) = (x$

$(a+b_1).x^3 + x^2.(4a-4b_1+b_2)$
$+x.(3a-5.b_1-4b_2)-5b_2$
$= (x+1)$

$a = -b_1$, $b_2 = \frac{-1}{5}$, $-8b_1 - \frac{1}{5} = 0$, $b_1 = \frac{-1}{40}$

$\frac{1}{40}.C_6 + C_5$

$\sim$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{40}z_3' & z_3' \end{bmatrix}$$

$a = -b_1$, $b_2 = \frac{-1}{5}$, $-8b_1 - \frac{1}{5} = 0$, $b_1 = \frac{1}{40}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{40} z_3' & z_3' \end{bmatrix}$$

b. $S_5 + S_6$

$\sim$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & x+1 & z_3' \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_1' & 0 \\ 0 & 0 & 0 & 0 & x+1 & z_3' \end{bmatrix}$$

$$(5-x).S_6 + S_5$$

$$\sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (5-x).z_3' \\ 0 & 0 & 0 & 0 & x+1 & z_3' \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (5-x)\cdot z_3' \\ 0 & 0 & 0 & 0 & x+1 & z_3' \end{bmatrix}$$

$S_5 \leftrightarrow S_6$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & (x+1) & z_3' \to 0 \\ 0 & 0 & 0 & 0 & 0 & (x-5)(x+1)\cdot(x+3)\cdot x \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x+1, & z_3' \\ 0 & 0 & 0 & 0 & \frac{1}{40} z_3' & z_3' \end{bmatrix}$$

$40 \cdot S_6 \sim S_6$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x+1 & z_3' \\ 0 & 0 & 0 & 0 & z_3' & 40 z_3' \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x+1 & z_3' \\ 0 & 0 & 0 & 0 & z_3' & 40\, z_3' \end{bmatrix}$$

$z_3' = x.(x+1)(x+3)$

$-x.(x+3)\,S_5 + S_6$

$\sim$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x+1 & z_3' \\ 0 & 0 & 0 & 0 & 0 & z_4 \end{bmatrix}$$

$z_4 = x.(x+1)(x+3)$

$(40 - x.(x+3))$

$z_4 = x.(x+1)(x+3).$

$\left[-(x^2+3x-40)\right]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x+1 & z_5' \\ 0 & 0 & 0 & 0 & 0 & z_4 \end{bmatrix}$$

$z_4 = x.(x+1)(x+3)$

$(1 - x.(x+3))$

$z_4 = x.(x+1)(x+3).$

$\left[-(x^2+3x-1)\right]$

$z_4' = x.(x+1).(x+3)$

$\left(x + \frac{3-\sqrt{13}}{2}\right)\left(x + \frac{3+\sqrt{13}}{2}\right)$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x+1 & 0 \\ 0 & 0 & 0 & 0 & 0 & z_4' \end{bmatrix}$$